On a possibility of adjoint colored states condensation at finite temperatures in lattice gauge model

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Cooled down and diluted quark-gluon matter is considered. A possibility of condensation of multiquark clusters with zero N-alities is discussed.

1. Introduction

Heavy ion experiments are still unable to produce QCD matter that is dense and hot enough to reveal an explicit evidence in favor of the presence of quark-gluon plasma. However, a prolonged intermediate period before hadronisation may give some indirect information about the transient initial stage.

The common feature of non-Abelian gauge theories is that the static potential between matter sources crucially depends on the corresponding representation of the gauge group [1]. $q\overline{q}$ states with zero and nonzero N-ality yield screening and confining potentials, respectively [2]. In this paper we assume that the retained forces at intermediate stage rearrange the uniform system and split it into clusters with zero N-ality. Colored states present $\left(N^2-2\right)/(N^2-1)$ part of the total number of $q\overline{q}$ states. Although eventually all multiquark states mandatory rearrange themselves into color singlets, the requirement that multiquark states must be color singlets may appear too restrictive at the intermediate stage. Lattice calculations [3,4] provide ample evidence that even at fairly high temperatures, color singlet objects propagate in plasma. Since the interaction between droplets with zero N-ality is the Debay-like interaction in the confined phase and Coulomb-like one in the deconfined phase, one may conclude that the gas of such clusters can't be regarded as ideal. Indeed, deviations from the ideal gas limit are found even at temperatures of about $5T_c$ [5].

Instead of SU(3) we use SU(2) which is expected to have very similar features. Moreover, many realistic models, e.g. flux tube models, do not distinguish between SU(2) and SU(3) [6].

2. Gaussian approximation for the effective action

Let us suppose we succeeded to integrate over spatial link variables U_n in the QCD action S and managed to express the effective action S_{eff} in terms of traces of Polyakov loop in fundamental representation $\chi_x = \mathrm{Tr_f}\left\{\Omega_x\right\}$. Then we assume that the Gaussian approximation

$$-S^{eff}\left(\chi_x\right) \simeq \eta_x\chi_x - \frac{1}{2}\chi_{x'}A_{x'-x}\chi_x, \qquad (1)$$

at least roughly, reflects the main features of critical behavior. The 'source' term $\eta_x\chi_x$ in (1) usually appears (after integration over matter fields) as a part of the effective fermion action

$$-S_F^{eff} \sim 2\sum_x\left(\eta\chi_x - M^2\chi_x^2\right). \qquad (2)$$

The 'mass' term $M^2\chi_x^2$ in (2) as well as the invariant measure contribution

$$\begin{aligned} d\mu_x &= \sqrt{(1-\chi_x^2/4)}\theta\left(4-\chi_x^2\right)d\chi_x/\pi \\ &\simeq e^{-\chi_x^2/8}\theta\left(4-\chi_x^2\right)d\chi_x/\pi \end{aligned} \qquad (3)$$

are to be included into the matrix $A_{x'-x}$. The compactness condition $\left(\chi_x^2<4\right)$ can be taken into account in a spherical model approximation

$$\prod_x\theta\left(4-\chi_x^2\right) \quad\to\quad \theta\left(4v-\sum_x\chi_x^2\right) \qquad (4)$$

$$= \int_{c-i\infty}^{c+i\infty} \frac{ds}{2\pi is} e^{4vs - s\sum_x \chi_x^2},$$

where $v = N_\sigma^3$ is the spatial lattice volume. This contribution adds to the $A_{x'-x}$ the complementary 'mass' term: $-s\sum_x \chi_x^2$, and the integration over χ_x can be easily done. To obtain the partition function, we integrate over s applying the saddle point method

$$2v^{-1}\ln Z \simeq 2s_0 - \ln\det A\left(s_0\right) + \eta_{x'} A^{-1}_{x'-x}\left(s_0\right)\eta_x,$$

where s_0 is the saddle point. Now the matrix $A^{-1}_{x'-x}$ can be related to the correlation function

$$A^{-1}_{x'-x} = \langle \chi_x \chi_{x'} \rangle - \langle \chi \rangle^2 \tag{5}$$

and expressed through the potential between probes taken from precision MC simulations

$$V_{1.1}^{(f)}\left(R\right)/T = -\ln\langle \chi_x \chi_{x'} \rangle = \alpha R - q/R - c, \tag{6}$$

where α is the string tension, and $R = |\mathbf{x} - \mathbf{x}'|$. The measured value of q is close to "IR charge" $q = \pi/12$. In the deconfinement region ($\alpha = 0$), one can put $c = -\ln\langle \chi_0 \chi_\infty \rangle = -\ln\langle \chi \rangle^2$.

The potential $V_{1.1}^{(A)}\left(|\mathbf{x} - \mathbf{x}'|\right)$ for $q\overline{q}$ in the adjoint state (for small $|\mathbf{x} - \mathbf{x}'|$)

$$V_{1.1}^{(A)} \simeq -T\ln\left(\left\langle\ \chi^2 \right\rangle - 1\right) \tag{7}$$

becomes to be complex for $\left\langle\ \chi^2 \right\rangle < 1$ – which means that the adjoint states are strongly suppressed in the corresponding parameter area.

Precision data [7] on $N_\sigma^3 \times 4$ lattices (N_σ = 12,18,26,36) show that

$$\langle\ |\chi|\ \rangle = 2B_{N_\sigma}\left(\beta/\beta_c - 1\right)^\varepsilon \tag{8}$$

with $\beta_c = 2.29895$, $\varepsilon = 0.327$. High statistics calculations allow us to take away corrections to scaling and to deduce [7] parameter $B_\infty \equiv \lim_{N_\sigma\to\infty} B_{N_\sigma} = 0.825(1)$ from finite volume data. Considering that (8) is in fair agreement with entire measured data (up to $\beta = 2.3$), one may assume that (8) gives a reasonable estimation for $\left\langle \chi^2 \right\rangle = \langle |\chi| \rangle^2 + O\left(1/v\right)$ in a wider area of β. In particular, we find that $\left\langle \chi^{(A)} \right\rangle$ becomes positive[1] for $\beta > 2.8$.

[1] The potential $V_{1.1}^{(A)}$ becomes negative for $\beta > 3.7$.

The potential for two ($\mathbf{x_1} \simeq \mathbf{x_1'} \simeq \mathbf{x}$ and $\mathbf{x_2} \simeq \mathbf{x_2'} \simeq \mathbf{x} + \mathbf{R}$) adjoint particles can be computed as

$$\begin{aligned} V^{(AA)}\left(R\right) &= F^{(AA)}\left(R\right) - F^{(AA)}\left(\infty\right) \\ &= -2\rho\left(R\right)q/R, \end{aligned} \tag{9}$$

where the function $\rho(R)$ slowly changes from 1 to $2\left\langle \chi \right\rangle^4 / \left\langle \chi^{(A)} \right\rangle^2$; $F^{(AA)}\left(R\right) \equiv -T\ln\left\langle \chi_0^{(A)} \chi_R^{(A)} \right\rangle$ and $F^{(AA)}\left(\infty\right) = -T\ln\left\langle \chi^{(A)} \right\rangle^2$. Therefore, for any pair of adjoint particles we get the attractive Coulomb-like potential.

3. Quasi-ideal gas of adjoint particles

To obtain the condensation condition, we consider a simple model where the energy of n adjoint particles is given by

$$E_n^{(A)} = E_{id}^{(A)} + V_n^{(A)}\left(x_1 ... x_n\right), \tag{10}$$

where $E_{id}^{(A)} = \sum_{k=1}^n \mathcal{E}_1\left(p_k; 2m\right)$, $\mathcal{E}_1(p;m) = \sqrt{p^2 + m^2} - m$ corresponds to the kinetic part of energy and $V_n^{(A)}\left(x_1 ... x_n\right)$ corresponds to the potential. Here we make use of the standard trick and, after the integration over p_k, write for the free energy $F \equiv -T\ln Z$

$$F = F_{id} - T\ln\left\{1 - v^{-n}\sum_{[x]}\left(1 - e^{-V_n^{(A)}/T}\right)\right\} \tag{11}$$

with $F_{id} = n\ \lambda\left(2m\right)$, where $\lambda\left(m\right)$ for $m \gg T$ is given by

$$\lambda\left(m\right) = \int e^{-\mathcal{E}_1(p;m)}\left(\frac{dp}{2\pi}\right)^3 \simeq \left(\frac{mT}{2\pi}\right)^{\frac{3}{2}}. \tag{12}$$

The gas is considered to be so diluted that the scattering of more than two adjoint particles may be neglected

$$V_n^{(A)}\left(x_1 ... x_n\right) \simeq \sum_{jk} V^{(AA)}\left(\left|\mathbf{x_j} - \mathbf{x_k}\right|\right), \tag{13}$$

where $V^{(AA)}\left(R\right)$ is given by (9), so

$$\sum_{[x]}\left(1 - e^{-V_n^{(A)}/T}\right) \simeq \tag{14}$$

$$n\left(n-1\right)\sum_{R > R_{\min}} e^{-2V_{1.1}^{(f)}(R)/T}$$

and, therefore, one can write

$$F \simeq F_{id} + n^2 TB/v; P = (1 + Bn/v)\,Tn/v, \quad (15)$$

where

$$B = -\sum_{R>R_{\min}}^{\sqrt[3]{3v/4\pi}} \left(e^{-V^{(AA)}(R)/T} - 1\right) \quad (16)$$
$$\sim -\frac{3\,\langle\chi\rangle^4\, v^{2/3}}{\left\langle\chi^{(A)}\right\rangle^2 T}.$$

Gas becomes unstable when $\partial P/\partial v \le 0$, which can be expressed in terms of the concentration as $n/v \ge Tv^{-2/3}\left\langle\chi^{(A)}\right\rangle^2 / \langle\chi\rangle^4$, so the condensation may start for a very diluted gas .

4. Area of adjoint states domination

Let us now try to estimate the value of β at which the formation of adjoint particles begins to dominate. With this in mind we compute the grand canonical partition functions $Z_f \equiv e^{-F^f/T}$ and $Z_A \equiv e^{-F^A/T}$ for the gas of fundamental and adjoint particles respectively. The energy for the fundamental particle gas is given by

$$E_{q;\overline{q}} = (q + \overline{q})\,\mathcal{E}_1(m) + V_{q;\overline{q}} \quad (17)$$

with $V_{q;\overline{q}}\left(x_1, ..., x_q, x'_{\overline{1}}, ..., x'_{\overline{q}}\right)$ for the potential energy of q quarks and $\overline{q}$ antiquarks. So, we get

$$e^{-F^f/T} = \sum_{q,\overline{q}=0}^{\infty} \frac{\lambda\,(m)^{\overline{q}+q}}{q!\overline{q}!} \sum_{[x;x']} e^{-V_{q;\overline{q}}/T}. \quad (18)$$

Now after [8], we may write

$$e^{-\frac{V_{q;\overline{q}}}{T}} = \mathrm{Tr}\left\{ \mathrm{e}^{-\mathrm{S}} \prod_{\mathrm{k}}^{\mathrm{q}} \chi_{\mathrm{x_k}} \prod_{\overline{\mathrm{k}}}^{\overline{\mathrm{q}}} \chi^*_{\mathrm{x}'_{\overline{\mathrm{k}}}} \right\} / \mathrm{Tr}\left\{\mathrm{e}^{-\mathrm{S}}\right\}$$
$$= \left\langle \prod_{k}^{q} \chi_{x_k} \prod_{\overline{k}}^{\overline{q}} \chi^*_{x'_{\overline{k}}} \right\rangle \quad (19)$$

or

$$e^{-F^f/T} = \left\langle \exp\left\{ \lambda \sum_x \left(\chi_x + \chi^*_x\right) \right\} \right\rangle. \quad (20)$$

Along the same line for the adjoin particles, we can get

$$e^{-F^A/T} = \left\langle \exp\left\{ \lambda\,(2m) \sum_x \chi_x^{(A)} \right\} \right\rangle. \quad (21)$$

To obtain a rough estimation for the parameter area where F^A becomes lower than F^f, we use the following approximation (instead of the Gaussian one): $\left\langle e^Q \right\rangle \simeq e^{\langle Q\rangle}$. If we compare

$$-\frac{F^A/v}{(2\pi)^{\frac{3}{2}}\sqrt{T}} \simeq (2m)^{\frac{3}{2}}\left(\langle|\chi|\rangle^2 - 1\right) \quad (22)$$
$$-\frac{F^f/v}{(2\pi)^{\frac{3}{2}}\sqrt{T}} \simeq 2\,(m)^{\frac{3}{2}}\langle|\chi|\rangle, \quad (23)$$

we conclude that $F^A < F^f$ in the area where F^A becomes negative, i.e. for $\langle|\chi|\rangle > \sqrt{2}$ or $\beta > 3.7$ for SU(2) (see also footnote in Section 2).

We considered a very simple model for cooled quark-gluon matter. It is shown that at $\beta > 2.8$ favorable conditions appear for the creation of Bose particles with zero N-ality. The formation of such clusters dominates at $\beta > 3.7$. Forces of attraction between such particles facilitate the condensation which may start even when a gas is very diluted.

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REFERENCES

1. P. H. Damgaard, M. Hasenbusch, Phys. Lett. **B331** (1994) 400.
2. G. Mack, DESY 77/58; Phys. Lett. **78B** (1978) 263.
3. C. E. DeTar, Phys. Rev. **D32** (1985) 276; *ibid* **37** (1987) 2328.
4. C. DeTar, J. Kogut, Phys. Rev. **D36** (1987) 2828.
5. G. Boyd et al., Nucl. Phys. **B469** (1996) 419.
6. M. Teper, Phys. Lett. **B397** (1997) 223.
7. J. Engels, T. Scheideler, Nucl. Phys. **B539** (1999) 557.
8. L. D. McLerran, B. Svetitsky, Phys. Rev.**D24** (1981) 450.